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རང་ཉིད་སློབ་སྟོན་མཁོ་ཆས།

Education in Emergency
Self - Instructional Materials

གནས་རིམ་༤ པ། སློབ་རིམ་༩-༡༠ པ།
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Key Stage 4 Cl - IX - X
Vol. V

Self-Instructional Materials

Key-stage IV

(Classes IX and X)

Published by

Ministry of Education in collaboration with the Royal Education Council

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TABLE OF CONTENTS

Lesson No: 1 **Subject:** Mathematics **Class:** 9 – 10 **Time:** 50 minutes

Topic: **Volume & Total Surface Area (TSA) of 3-D Shapes**

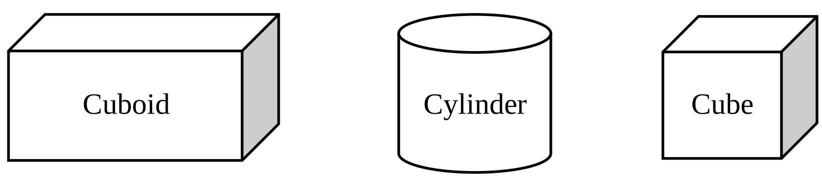
1. Define volume and total surface area.
2. Calculate the volume and total surface area of any 3-D shapes.
3. Solve the problems related to 3-D shapes.

Introduction

3-D Shapes/Three-Dimensional Shapes

Shapes that have three dimensions, namely length, width, and height are called **3-D shapes**.

These are some of the 3-D shapes:

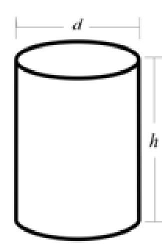


Volume

The volume of an object is the amount of space occupied by an object, which is three dimensional in shape. It is often measured in cubic units such as cm^3 and m^3. In other words, the volume of any object or container is the capacity of the container to hold the amount of fluid (gas or liquid). The volume of cube, cuboid or rectangular prism, cylinder, prism, and cone can be easily calculated by using formulas.

For example, for a cylinder, the volume of the cylinder can be measured by using the formula $\pi r^2 h$ where,

- π is a constant approximately equal to 3.14 or $\frac{22}{7}$
- r = radius of the circular base.
- h = height of the cylinder

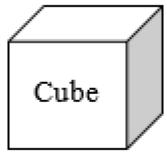


Note: d = diameter of the circular base

(If diameter is given then radius is half of diameter $r = \frac{d}{2}$)

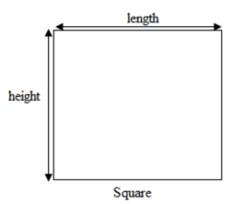
Total Surface Area (TSA or SA)

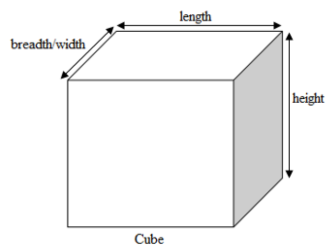
The surface area is the sum of the areas of all the faces and curved surfaces of a 3-D shape. It is measured in square units such as cm^2 and m^2.
For example, in a cube total surface area is the area of the 6 faces (square face).
i.e. **TSA or SA of cube = $6s^2$,** where 6 is the total number of faces in a cube (all square face) and s is the side length of the face (s^2= area of a square, since each face of the cube, is square).



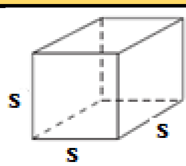
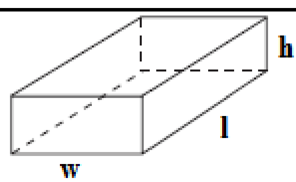
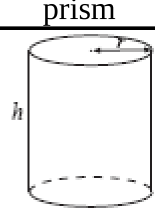
Note:

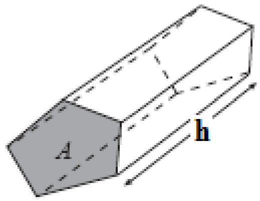
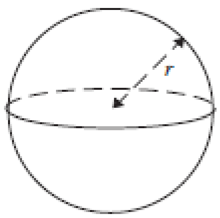
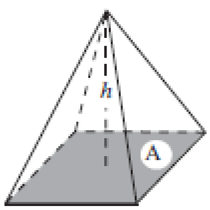
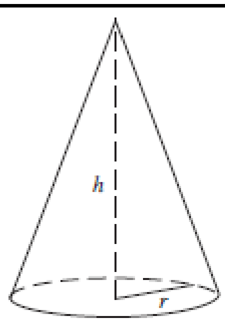
'Area' or 'surface area' of a figure means exactly the same thing. So 'total surface area' simply means 'total area of the figure, regardless of whether the figure is a 2-D figure (such as a square) or a 3-D figure (such as a cube).





Some of the formulas to find out volumes & total surface areas of basic shapes are:

Shapes	Volume & SA Formula	Variables
 Cube	Volume = s^3 SA = $6s^2$	s = length of edge or side
 Cuboid/rectangular prism	Volume = $l \times w \times h$ SA = $2(lw + wh + lh)$	l = Length, w = Width, h = Height
 Cylinder	Volume = $\pi r^2 h$ Area of curved surface = $2\pi r h$ Area of each end = πr^2 SA = $2\pi rh + 2\pi r^2$	r = radius of the circular edge, h = height

 Prism	Volume =Ah SA=2A+hP	A = area of base, h = height p = perimeter of base
 Sphere	V = $\frac{4}{3}\pi r^3$ SA=4 πr^2	r = radius of the sphere
 Pyramid	V = $\frac{1}{3}$ Ah SA=A + Area of lateral faces	A = area of the base, h = height of the pyramid
 Cone	V = $\frac{1}{3}\pi r^2 h$ SA= $\pi r^2 + \pi rs$	r = radius of the circular base, h = height (base to tip) s = slant height of cone (which is hypotenuse)

Note (s): Volume Versus Capacity

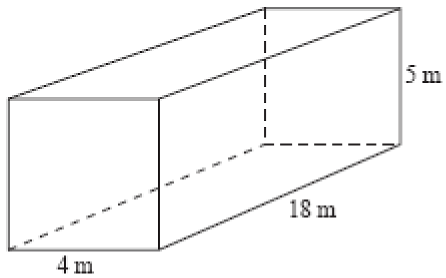
Volume is the space taken up by the object itself, while capacity refers to the amount of substance, like a liquid or a gas, that a container can hold.

(use this note for question 3 (b) of **self-check** of this lesson)

Capacity	Volume
1 mL	1 cm^3
1 L	1000 cm^3
1000 L	1 m^3

Some Solved Examples

Example 1



a. Calculate the *volume* of the cuboid shown.

Solution:

Volume = l × w × h

=18 x 4 x 5

= 360 m^3

Therefore, the volume of the cuboid is 360 m^3.

b. Calculate the *surface area* of the cuboid shown.

Solution:

SA = 2(lw + wh + lh)

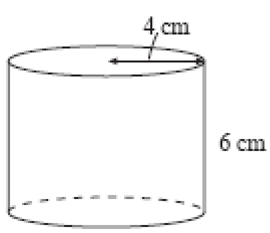
= 2 (4 × 18 + 4 × 5 + 5 × 18)

= 2 (182)

= 364 m^2

Therefore, the SA of the cuboid is 364 m^2.

Example 2



Calculate the *volume* and total *surface area* of the cylinder shown.

Solution:

Volume = $\pi r^2 h$

= π x 4^2 x 6

= π x 16 x 6

= 96π (π = 3.14159265358979 ≈ 3.14)

= 301.5928946 cm^3

= 302 cm^3 (to 3 significant figures)

Therefore, the volume of the cylinder is 302 cm^3.

Area of curved surface = $2\pi rh$

= 2 x π x 4 x 6

= 48π

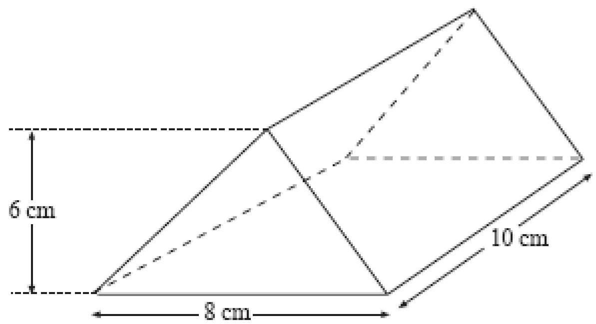
= 150.8 cm^2

Area of each end = πr^2
$= \pi \times 4^2$
$= 16 \times \pi$
$= 50.3 \text{ cm}^2$

TSA/SA = $2\pi rh + 2\pi r^2$
$= 150.8 + (2 \times 50.3)$
$= 150.8 + 100.6$
$= 251.4 \text{ cm}^2$
$= 251 \text{ cm}^2$ (to 3 significant figures)

Therefore, the total surface area (TSA/SA) is 251 cm^2.

Example 3



Calculate the *volume* of this prism.
Solution:
Volume =Ah (A is an area of base)

Area of base = ½ x b x h
= ½ x 8 x 6
= 24 cm^2

Volume =Ah
= 24 x 10
= 240 cm^3
Therefore, the volume of the prism is 240 cm^3.
(As per the question, no need to calculate SA)

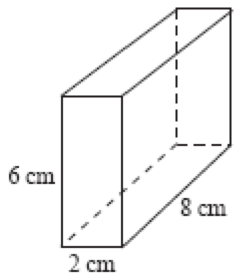
Activity 1

Solve the questions given below in your notebook.

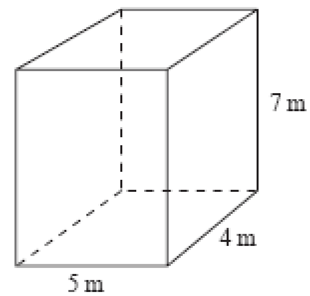
Question 1

Calculate the *volume* and *surface area* of each of the following cuboids:

(a)



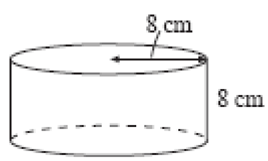
(b)



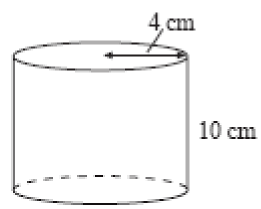
Question 2

Giving your answers correct to 3 significant figures, calculate the *volume* and *total surface area* of each of the following cylinders:

(a)



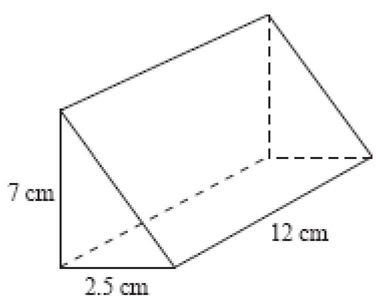
(b)



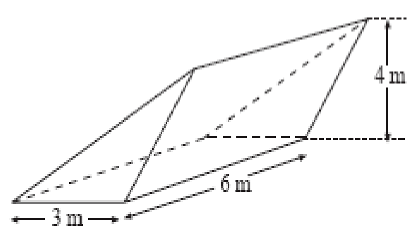
Question 3

Calculate the *volume* of each of the following prisms:

(a)



(b)



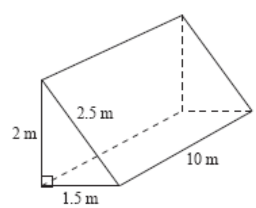
Summary

- Shapes that have three dimensions, namely length, breadth and height are called **3-D shapes**.
- The volume of an object is the amount of space occupied by the object.
- It is usually measured in terms of cubic units.
- In other words, the volume of any object or container is the capacity of the container to hold the amount of fluid (gas or liquid).
- Surface area is the sum of the area of all the faces and curved surfaces of a 3-D shape.
- It is usually measured in square units.

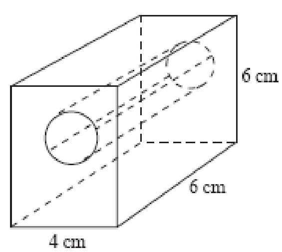
Self-check for Learning

Solve the questions given below in your notebook.

1. Calculate the volume and surface area of the prism.



2. The diagram shows a wooden block that has had a hole drilled in it. The diameter of the hole is 2 cm.



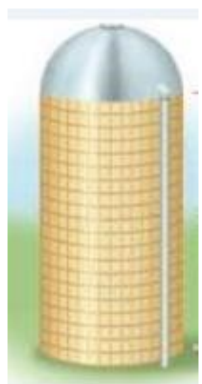
Calculate the *volume* of this solid, giving your answer correct to 2 decimal places. ***Hint:*** *Volume = Volume of block - Volume of hole*

Self-check for Learning (Continued)

Solve the questions given below in your notebook.

3. A cone shaped paper cup has a diameter 7cm and depth of 12 cm. (Assume $\pi = \frac{22}{7}$)
   a. What is the area of the paper need to make the cup?
   b. What is the capacity of the cup?

4.
   a. A farmer keeps grains in a silo with a hemisphere roof. Calculate the number of square meters that would need to be painted on a silo if the diameter was 3.8m and the total height to the top of the hemisphere was 9.3m.



   b. Estimate the volume of grain that this silo could hold.

Self-check for Learning

1. Volume = 15 m^3, SA = 63 m^2

2. Volume = 125.15 cm^3

3.
a. 176 cm^2
b. 154 mL

4.
a. 111.1 m^2
b. 98.3 m^3

Lesson No: 2 **Subject:** Mathematics **Class:** 9 – 10 **Time:** 50 minutes

Topic: **Equation of Linear Graph**

1. State the meaning of a linear graph.
2. Define an equation.
3. Calculate the problems related to linear graph.

Introduction

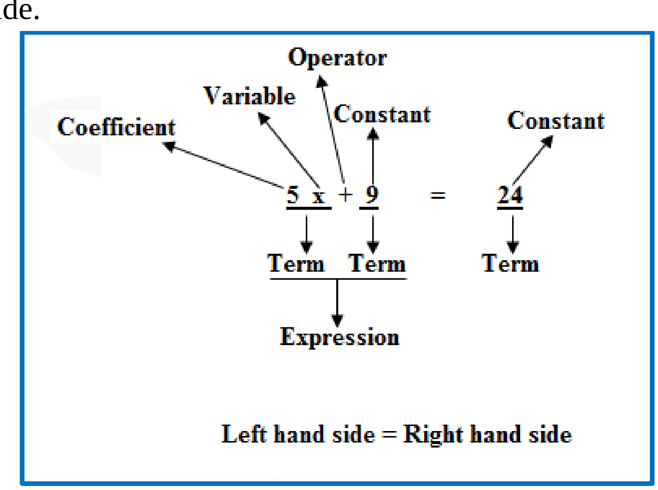
Linear Graph

Linear means straight and a **graph is** a diagram that shows a connection or relation between two or more quantity. So, the **linear graph** is nothing but a **straight line** or **straight graph** which is drawn on a plane connecting to points on x and y coordinates. We use linear relations in our everyday life and by graphing those relations in a plane, we get a straight line.

Equation

A mathematical statement consisting of an equal symbol between two algebraic expressions that have the same value is called an **equation.**
The most basic and common algebraic equations in math consist of one or more variables.
For instance, $3x + 5 = 14$ is an equation, in which $3x + 5$ and 14 are two expressions separated by an 'equal' sign.

In an algebraic equation, the left-hand side is equal to the right-hand side.
Here, for example, $5x + 9$ is the expression on the left-hand side, which is equal to the expression 24 on the right-hand side.



The process of finding the value of the variable is called solving the equation.

Note (s):

- *2x + 17y – 3* is not an equation, because it does not consist of the equals sign. It is only an expression.
- The study of algebra is largely about learning to solve various kinds of equations.

Equation of Linear Graph/Linear Graph Equation

As discussed above, linear graph forms a straight line and denoted always as an equation;

$$y = mx + b$$

where **m** is the slope of the graph and **b** is the y-intercept of the graph.

The value of slope (**m)** is the ratio of the difference of y-coordinates and the difference of x-coordinates.

$$\text{i.e. } m = \frac{y_2 - y_1}{x_2 - x_1} \textbf{ OR } \frac{rise}{run}$$

The linear equation can also be written as,

***ax* + *by* + *c* = *0* (General/Standard Form)**

where *a, b* and *c* are constants

Linear Graph Examples

Let us understand the linear graph definition with examples.

1. The equation *y=2x+1* is a linear equation or forms a straight line on the graph. When the value of x increases then ultimately the value of y also increases by twice the value of *x* plus 1.
2. Suppose, if we have to plot a graph of a linear equation *y=2x+1*.

Let us consider *y=2x+1* forms a straight line. Now, first, we need to find the coordinates of x and y by constructing below the table;

x	-2	-1	0	1	2
y					

Now calculating value of y with respect to x, by using given linear equation,

y = 2x + 1

y = 2(-2) + 1 = -3 for x = -2

y = 2(-1) + 1 = -1 for x = -1

y = 2(0) + 1 = 1 for x = 0

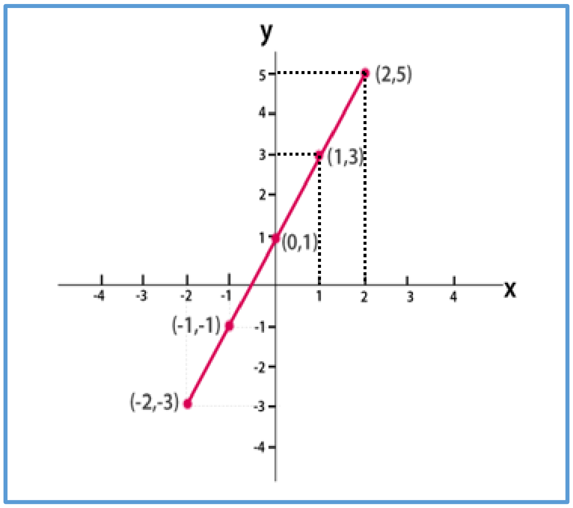
y = 2(1) + 1 = 3 for x = 1

y = 2(2) + 1 = 5 for x = 2

So, the table can be re-written as;

x	-2	-1	0	1	2
y	-3	-1	1	3	5

Now based on these coordinates we can plot the graph as shown below.



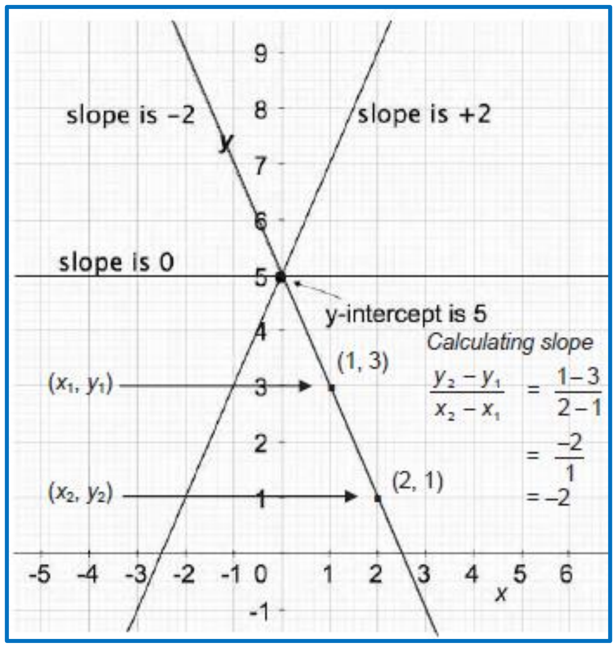
The Meaning of Slope and *Y*-Intercept

The **slope** of a line tells the rate at which ***y*-variable** changes in terms of the ***x*-variable**:

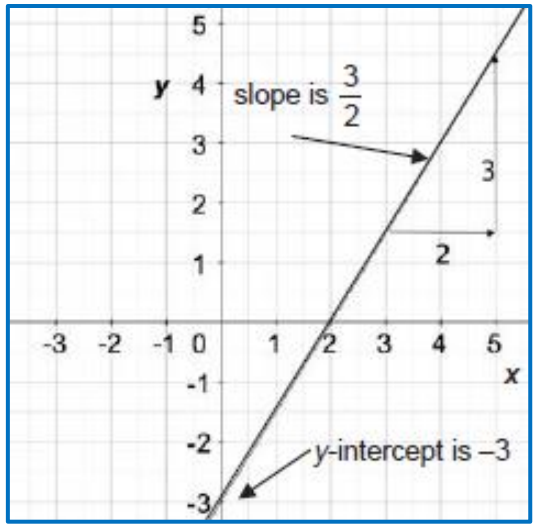
- If the slope is positive, *y*-variable increases as the *x*-variable increases.
- If the slope is negative, *y*-variable decreases as the *x*-variable increases.
- If the slope is 0, the *y*-variable does not change as the *x*-variable changes.
- The slope (m) can be calculated using any two points on the line:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- The ***y*-intercept** is the value of the *y*-coordinate where the line meets or crosses the *y*-axis.



Slope and *Y*-Intercept Form



When an equation of a line is in the form $y = mx + b$, it is called the ***y*-intercept form** of the equation.

- ❖ The value of *m* is the slope of the line.
- ❖ The value of *b* is the *y*-intercept.

If you know the slope of a line and its *y*-intercept, you can use this information to write an equation for the graph.

For example, this graph has a slope of $\frac{3}{2}$ and the *y*-intercept of –3, so, the equation of the line is $y = \frac{3}{2}x - 3$.

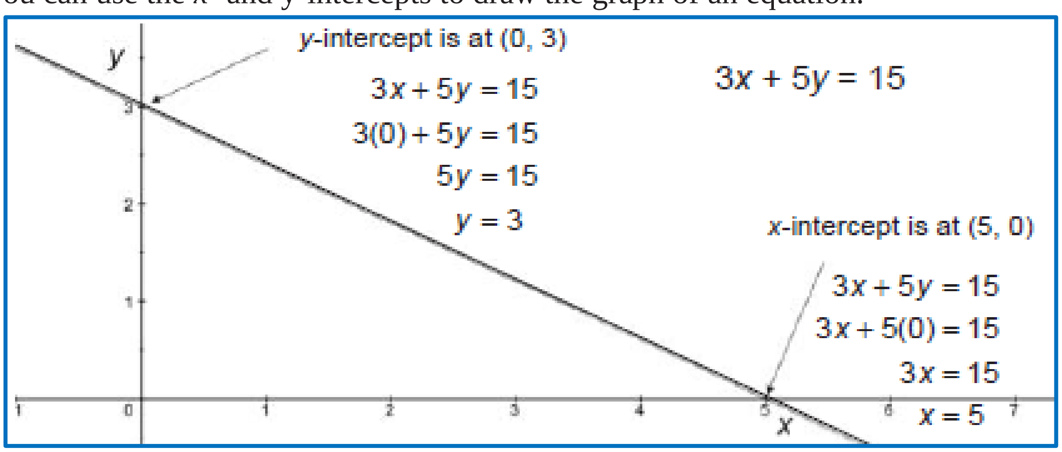
Standard Form

One form for writing the equation of a line is the slope and *y*-intercept form that you have already used. Another form is called the standard form. When an equation for a line is in the form **Ax + By = C**, it is said to be in **standard form**.

You can easily determine both the *x*- and *y*-intercepts of the graph from the standard form of the equation:

- ❖ The *y*-intercept is the value of the *y*-coordinate where the line meets or crosses the *y*-axis. The coordinates of the *y*-intercept look like (0, *y*). You can substitute $x = 0$ into the equation and solve it to determine the *y*-intercept.
- ❖ The ***x*-intercept** is the value of the *x*-coordinate where the line meets or crosses the *x*-axis. The coordinates of the *x*-intercept look like (*x*, 0). You can substitute $y = 0$ into the equation and solve it to determine the *x*-intercept.

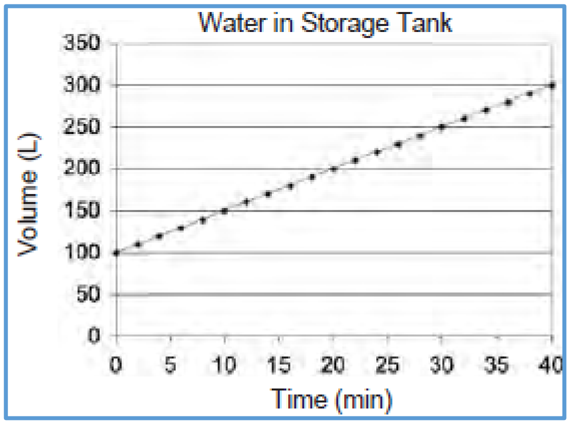
- You can use the *x*- and y-intercepts to draw the graph of an equation.



Some Solved Examples

Example 1: Interpreting Slope and *Y*-intercept

This graph shows the volume of water in a storage tank as it was being filled.



a) How much water was in the tank initially?

b) At what rate was the tank being filled?

c) How are the values in **part a)** and **part b)** related to the graph?

Solution:

a) The tank started with 100 L of water.

b) The tank was filled at a rate of 5 L/min.

c) 100 L is the *y*-intercept of the graph. 5 L/min is the slope.

Explanation

a) The graph shows that at 0 min there were 100 L of water in the tank.

b) I knew I could use any two points on the graph to calculate the rate at which the tank was filled because the rate is the same as the slope.

• I used the points at 0 min (0, 100) and at 10 min (10, 150) to find the slope:

$$\frac{150-100}{10-0} = \frac{50}{10} = 5$$

Example 2: Predicting Slope and *Y*-intercept

An empty water barrel weighs 15 kg. It is filled with water at a rate of 0.5 L/s.
The mass of 1 L of water is 1 kg. Suppose you drew the graph of the combined mass of the barrel and water as it is filled.

a) How do you know the graph will be a straight line?

b) What will be the slope and *y*-intercept of the graph?

Solution:

a) The mass of the barrel and water changes at a constant rate so the graph must be a straight line.

b) The slope is 0.5.
The *y*-intercept is 15.

Explanation

a) I knew 0.5 L of water went into the barrel every second. Since 1 L of water is 1 kg that meant the mass increased by 0.5 kg each second.

b) A rate of 0.5 L/s means that, for a run of 1 s, the rise would be 0.5 kg. That's a slope of $\frac{0.5}{1} = 0.5$.

• I knew that y-intercept was the value of the y-coordinate when x = 0. At 0 s, there was no water in the barrel, so the mass was the mass of the empty barrel, which was 15 kg.

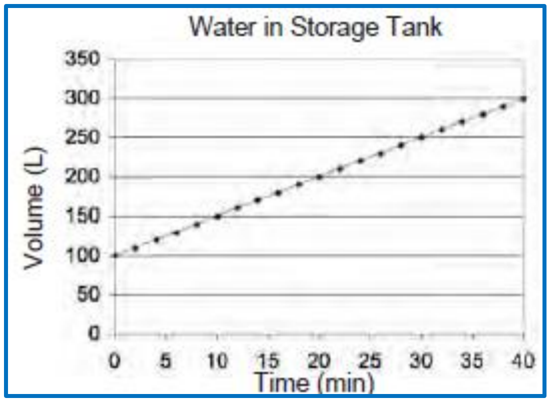
Example 3: Determining the Equation of a Line Given its Graph

The graph shows the volume of water in a storage tank as the tank was being filled over a 40 min period.

Write an equation for this graph.

Solution:

The equation of the line is $y = 5x + 100$.



Explanation

- I could see from the graph that the y-intercept was 100.
- I calculated the slope by dividing the rise by the run using the values at 40 m (40, 300) and at 0 m (0, 100):

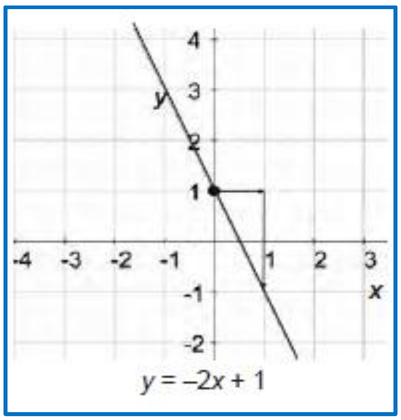
$$\frac{300-100}{40-0} = \frac{200}{40} = 5$$

- So, I knew that m = 5 and b = 100 and I substituted these values into y = mx + b.

Example 4: Sketching the Graph of a Line Given its Slope and *Y*-intercept

Sketch the graph of a line that has a slope of –2 and a *y*-intercept of 1. Then write the equation for the graph.

Solution:



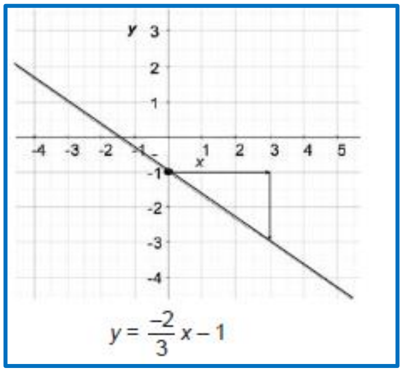
Explanation

- I marked the y-intercept at the point (0, 1) on the y-axis. It's easier to plot the y-intercept before the slope since there is only one spot it can be.
- Since the slope is –2, I knew that, for a run of 1, there had to be a rise of –2. A rise of –2 means the graph goes down.
- I marked a point that was 1 unit to the right (the run) and 2 units down (the rise) from the y-intercept.
- Then I drew a line through the two points.

Example 5: Sketching a Line Given its Equation

Sketch the graph of a line represented by the equation $y = -\frac{2}{3}x - 1$.

Solution:



Explanation

- The equation was in the form $y = mx + b$, so I knew the slope (m) was $-\frac{2}{3}$ and the y-intercept (b) was –1.
- I first plotted the y-intercept on the y-axis at (0, –1).
- The slope was negative, so I knew the line had to go down from left to right.
- I plotted a point that was 3 units to the right (a run of 3) and 2 down (a rise of -2) from the y-intercept. Then I joined the two points with a line.

Example 6: Graphing Given the Equation in Standard Form

Dawa used the equation $0.05x + 0.02y = 2000$ to represent how many 5% bonds (x) and 2% investment certificates (y) he would invest in to earn Nu 2000.

a) Sketch the graph that shows the possible combinations of bonds and certificates that could be invested to earn Nu 2000.

b) Use the graph to determine two possible combinations that will earn Nu 2000.

Solution:

a) $0.05(0) + 0.02y = 2000$

$0.02y = 2000$

$y = 2000 \div 0.02$

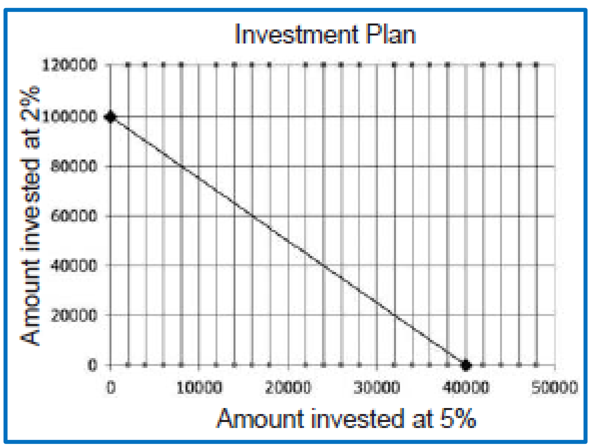
$= 100{,}000$

$0.05x + 0.02(0) = 2000$

$0.05x = 2000$

$x = 2000 \div 0.05$

$= 40{,}000$



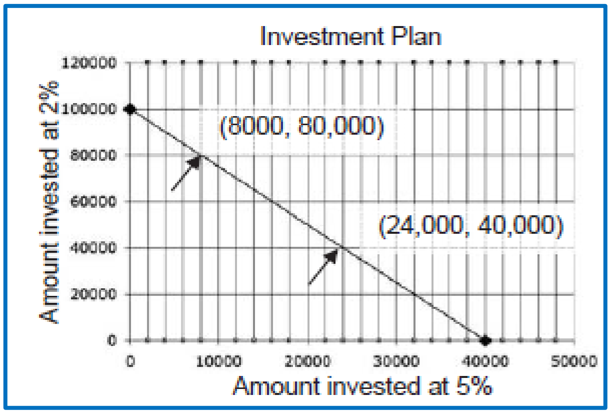
Explanation

a) I knew if I plotted the x- and y-intercepts, I could just join them to draw the graph. The equation was in the standard form **Ax + By = C** so it was easy to figure them out:

- ✓ The y-intercept occurs when x = 0, so I substituted x = 0 into the equation and solved for *y*.
- ✓ The x-intercept occurs when y = 0, so I substituted y = 0 into the equation and solved for x.
- ✓ I marked these points on the axes and then joined them to draw the graph.

b) I chose two points on the graph that had coordinates that were easy to read because they were on the intersection of grid lines.

b)



Example 7: Writing an Equation to Describe a Situation

Yuden wants to invest her money so it will earn interest.

- She will deposit some of her money with a bank that pays 4.2% interest.
- The rest she will use to buy a stock that is currently paying dividends of 9.6%.

She wanted to earn Nu 400 from this investment combination.
Write an equation to describe her investment plan.

Solution:

Let b represent the amount she invests with the bank and s represent the amount Yuden invests in stocks. Her investment plan is described by the following equation:

$0.042b + 0.096s = 400$

Explanation

✓ I knew I needed one variable for the amount deposited in the bank and another for the amount invested in stocks.

✓ She hopes to earn 4.2% on her bank deposit, so I knew I had to multiply that investment amount by 0.042.

✓ She hopes to earn 9.6% on stocks, so I had to multiply that investment amount by 0.096.

Example 8: Determining Slope and *Y*-intercept Form Given Standard Form

Determine the slope and y-intercept form of the line with equation $5x + 2y = 10$.

Solution 1:

$5x + 2y = 10 \rightarrow y = mx + b$

$5x + 2y = 10$

$5x - 5x + 2y = 10 - 5x$

$2y = 10 - 5x$

$2y = -5x + 10$

$\frac{2y}{2} = \frac{-5x+10}{2}$

$y = \frac{-5}{2}x + \frac{10}{2}$

$y = \frac{-5}{2}x + 5$

Explanation

I rearranged the equation so y was by itself on the left.

✓ I subtracted 5x from both sides.

✓ I knew that $10 - 5x = -5x + 10$.

✓ I divided both sides by 2.

✓ I knew that $\frac{-5x+10}{2} = \frac{-5}{2}x + \frac{10}{2}$

✓ I simplified $\frac{10}{2}$ to 5.

Solution 2:

To determine the y-intercept, set $x = 0$ and solve for y.

$$5(0) + 2y = 10$$
$$2y = 10$$
$$y = 5$$

The y-intercept is 5.

So, in $y = mx + b$ form of the equation, b = 5.

To determine the x-intercept, set $y = 0$ and solve for x.

$$5x + 2(0) = 10$$
$$5x = 10$$
$$x = 2$$

The x-intercept is 2.

Slope = m = $\frac{y_2-y_1}{x_2-x_1} = \frac{5-0}{0-2} = \frac{-5}{2} = -\frac{5}{2}$

The equation of the line: $y = -\frac{5}{2}x + 5$.

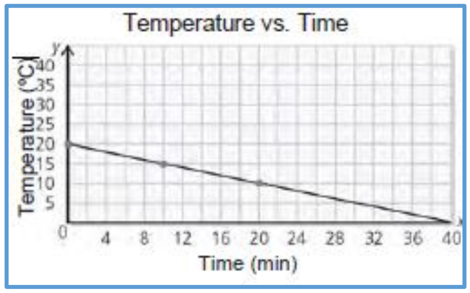
Explanation

- ✓ I knew I needed the slope and the y-intercept to write the equation.
- ✓ I used the standard form of the equation to determine the y-intercept by setting x = 0.
- ✓ I determined the x-intercept using the equation by setting y = 0.
- ✓ I used the coordinates of the intercepts to calculate the slope.
- ✓ I used the value of the slope and the y-intercept to write the equation.

Activity 1

Solve the questions given below in your notebook.

This graph shows the temperature of water in a glass.



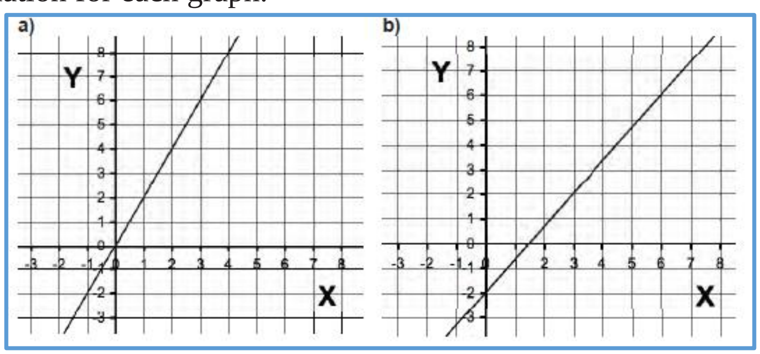
a. What does the slope of the graph represent?

b. The slope is negative. What does that tell you about what happened to the water temperature?

c. What does the *y*-intercept represent?

d. If the water-cooled at the same rate but the *y*-intercept had been 25, how would that have changed the graph?

2. Determine the slope and *y*-intercept for each graph. Then write the equation for each graph.



3. Sketch the graph of each line below. Then write its equation.

   a. the slope is 1 and y-intercept is –1

   b. the slope is –3 and y-intercept is 1

4. A line has equation $3x + 2y = 6$.

   a. Determine the coordinates of the *y*-intercept.

   b. Determine the coordinates of the *x*-intercept.

   c. Calculate the slope of the line.

   d. Write the slope and *y*-intercept form of the equation.

Summary

- **Linear** means straight and a **graph i**s a diagram which shows a connection or relation between two or more quantity.
- The **linear graph** is nothing but a **straight line** or **straight graph.**
- A mathematical statement consisting of an equal symbol between two algebraic expressions that have the same value is called an **equation.**
- In an algebraic equation, the left-hand side is equal to the right-hand side.
- The process of finding the value of the variable is called solving the equation.
- Linear graph forms a straight line and denoted always as an equation; $y = mx + b$.
- The **slope** of a line tells the rate at which the *y*-variable changes in terms of the *x*-variable:
- When an equation of a line is in the form $\boldsymbol{y = mx + b}$, it is called the ***y*-intercept form** of the equation.
- When an equation for a line is in the form $\mathbf{A}\boldsymbol{x} + \mathbf{B}\boldsymbol{y} = \mathbf{C}$, it is said to be in **standard form**.
- The ***y*-intercept** is the value of the *y*-coordinate where the line meets or crosses the *y*-axis.
- The ***x*-intercept** is the value of the *x*-coordinate where the line meets or crosses the *x*-axis.

Self-check for Learning

Solve the questions given below in your notebook.

1. In many cities, when you hire a taxi you pay a base amount plus a rate per minute of travel time.
   a) Suppose you were to graph the relationship between the time it takes to complete a trip and its cost. Why would the graph be linear?
   b) What would the slope represent?
   c) What would the *y*-intercept represent?

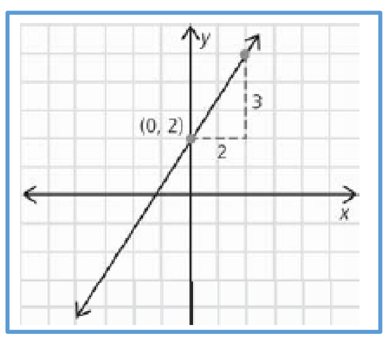
2. Sketch each graph.
   a) $y = \frac{1}{2}x - 2$ b) $y = -\frac{2}{3}x + 3$

3. Karma wrote a multiple-choice test. The scoring system worked as follows:

•gain 4 points for each correct answer •lose 1 point for each incorrect answer •0 points for unanswered questions. Karma received 60 points on the test.

   a) How does the equation $4c - i = 60$ describe all the different combinations of correct and incorrect answers Karma could have had to get a score of 60?
   b) Use *x*- and *y*-intercepts to graph this equation.
   c) What does each intercept mean? Are both intercepts possible? Explain.
   d) If Karma got the same number of questions correct as incorrect, how many questions did he answer on the test?

4.



   a) Write the equation of the graph above in slope and *y*-intercept form and in standard form.
   b) Use your answer from **part a)** to determine points on the line that have integer coordinates.
   c) Graph $3x - 2y = -4$
   d) How does your graph in **part c)** compare to the graph above? Why do you think this happened?

Self-check for Learning

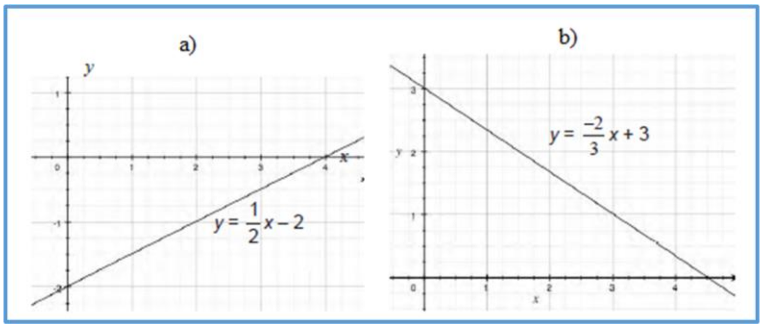
1.

a) The cost changes at a constant rate.

b) Cost per minute above base amount

c) Base amount

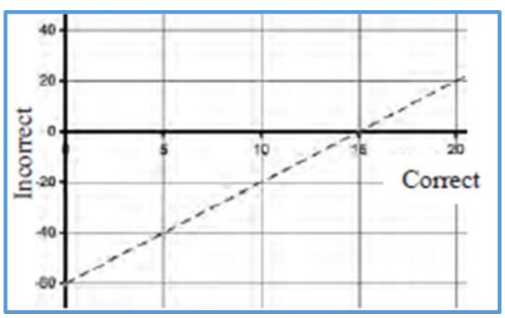
2.



3.

a) c is number of correct answers and i is number of incorrect answers; so, add $4c$ points for correct answers & subtract i points for incorrect answers, which is $4c - i$ points; final score is 60 points so $4c - i = 60$.

b) x-intercept is 15; y-intercept is – 60.



c) x-intercept means 15 correct answers with 0 errors; y-intercept means – 60 errors and 0 correct answers; a y-intercept of – 60 is not possible because you cannot have a negative number of answers.

d) 40

Self-check for Learning (Continued)

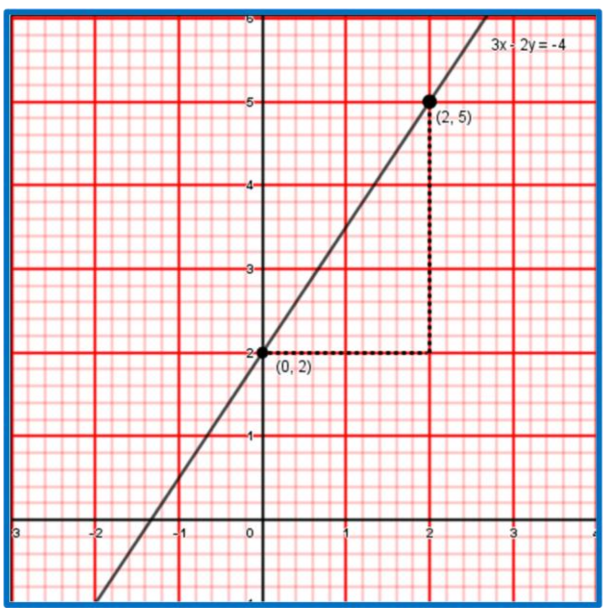
4.

a) $y = \frac{3}{2}x + 2$ *and* $\frac{3}{2}x - y = -2$

b) Sample response:

(0,2), (2,5), (-2, -1), (4,8)

c)



d) Same graph as $\frac{3}{2}x - y = -2$ because $\frac{3}{2}x - y = -2$ and $3x - 2y = -4$ are equivalent equations.

Lesson No: 3 **Subject:** Mathematics **Class:** 9 – 10 **Time:** 50 minutes

Topic: **Operation with Polynomials**

1. Identify the types of polynomials based on order of degree and number of terms.
2. Model with algebra tiles to add and subtract polynomials using zero principle.
3. Add, subtract, multiply and divide symbolically.
4. Multiply and divide polynomials using algebra tiles.

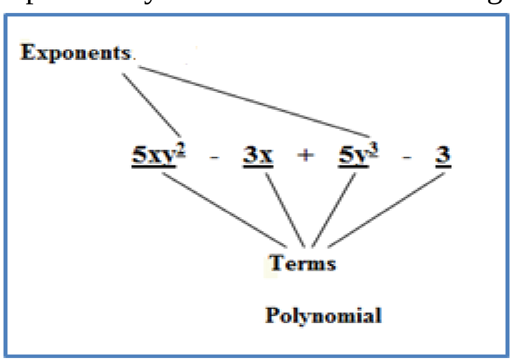
Introduction

Polynomial

A **polynomial** is an algebraic expression that includes at least one variable. It usually involves numbers and operation signs as well. The variable in the expression can be raised to one or more whole number powers, for example, $x + x^2$. The variables cannot be raised to any fractional or negative power. For example, $5x^{-2}$ and $5x^{-\frac{1}{2}}$ are not polynomials.

Types of Polynomials based on the Number of Terms

Each part of the polynomial separated by addition or subtraction signs is called a **term**.



Polynomials with 1, 2, or 3 terms have special names.

Number of Terms	Type of Polynomials	Examples
1	Monomial	$x or\ 5\ or\ 2y\ or\ xy$
2	Binomial	$2x + 2y\ or\ x + 4$
3	Trinomial	$2x + 2y - xy\ or\ x - y + x^2$

The **degree** of the polynomial is determined by the highest power. For example:

- The binomial $5x^2 - 3x$ is of degree 2 since the highest power of x is 2.
- The trinomial $2 - 3m - m^3$ is of degree 3 since the highest power of m is 3.

- The monomial xy appears to be of degree 1 since there is no exponent. However, it is actually of degree 2. You add the degrees of the two variables, x (which is x^1) and y (which is y^1) because the variables are multiplied.
- The binomial $2x + y$ is of degree 1 since each variable is of degree 1 and no variables are multiplied together.

Types of Polynomials based on the degree

Order of Degree	Type of Polynomials	Examples
0	Constant	6 or $-\frac{2}{3}$
1	Linear	$x + 2$ *or* $-\frac{3}{4}x - 5$
2	Quadratic	$3x^2 - 5x + 8$ *or* $5y^2 - \frac{1}{4}$
3	Cubic	$2x^3 - 6x^2 + 5x + 6$ *or* $2y^3$

Numerical Coefficient

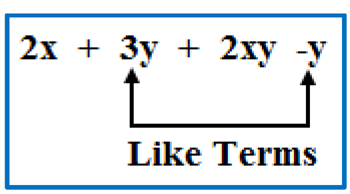
When a variable is multiplied by a number, whether an integer, a fraction, a decimal, or an irrational number like π, that number is called a **numerical coefficient**.
For example, 4 is a coefficient of x^2 in the monomial $4x^2$. If the coefficient is 1, it is usually not written; for example, instead of 1y, it is more common to write y.

Variable and Constant Terms

Polynomials can have **variable terms** and **constant terms**.
For example, in the binomial $2y + 8$, $2y$ is the variable term and 8 is the constant term.

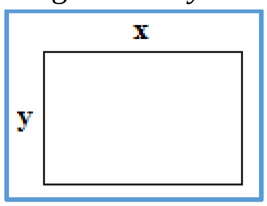
Like Terms

Sometimes polynomials include **like terms**. These are terms involving exactly the same variables raised to exactly the same powers. Any other pair of terms would be called, **unlike terms**.
For example, in the polynomial below, the terms $3y$ and y are like terms. Notice that $2x$ and $2xy$ are unlike terms because, even though both contain a 2 and an x, the variable parts of the terms are not identical.



A polynomial can only be classified after all the like terms have been combined.
For example, $2x + 3x$ may appear to be a binomial, but since you can combine the two like terms to create $5x$, it is actually a monomial. Similarly, $2x - 5x + 13$ can be simplified to $-3x + 13$, making it a binomial. Combining like terms is a way to **simplify** the polynomial expression.

- The terms of a polynomial are sometimes described by considering the operation signs in front of them. For example, in the trinomial $3x^2 + 2xy - 3$, the third term is generally considered to be -3 and not 3.
- You can **evaluate** a polynomial by substituting a value for the variables. For example, to evaluate $3x - 4$ when $x = 2$, calculate $3(2) - 4 = 2$.
- Polynomials can be used to represent certain situations. For example, consider the rectangle below with side lengths x and y.



Some polynomials related to this rectangle are:

- x to describe one dimension
- y to describe the other dimension
- $2x + 2y$ to describe the perimeter
- xy to describe the area

Standard Form

The Standard Form for writing a polynomial is to put the terms with the highest degree first.

Example:

Put this in Standard Form: $3x^2 - 7 + 4x^3 + x^6$

The highest degree is 6, so that goes first, then 3, 2 and then the constant last:

$$\mathbf{x^6} + 4\mathbf{x^3} + 3\mathbf{x^2} - 7$$

You **don't have to** use Standard Form, but it helps.

Operations with Polynomials

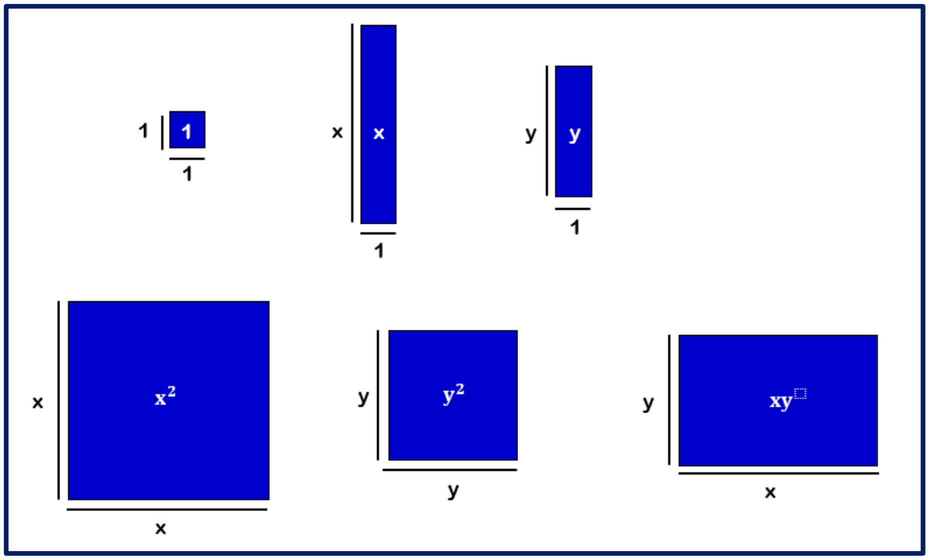
Operations of polynomials can be done in two ways:

1. Representation of algebra tiles (using an algebraic tile)
2. Symbolical interpretation (symbolically)

Describing algebra tiles

One of the ways to model or represent polynomials is by using algebra tiles. These are rectangular and square tiles with particular lengths and widths as shown below.

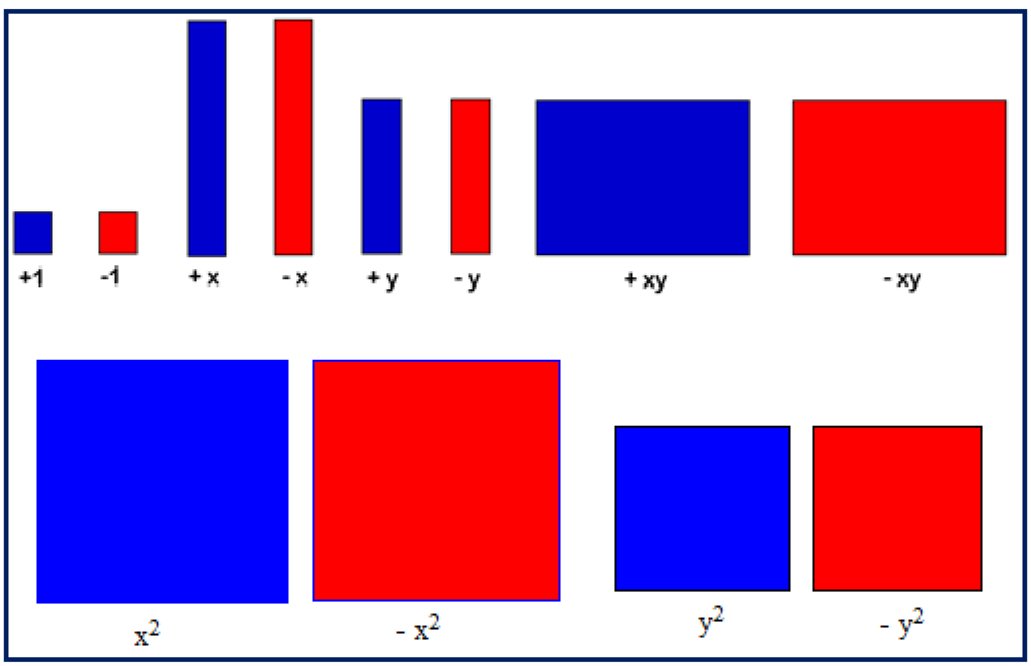
(The **algebra tiles** are shown on the next page)



Using Algebra tiles to represent Polynomials

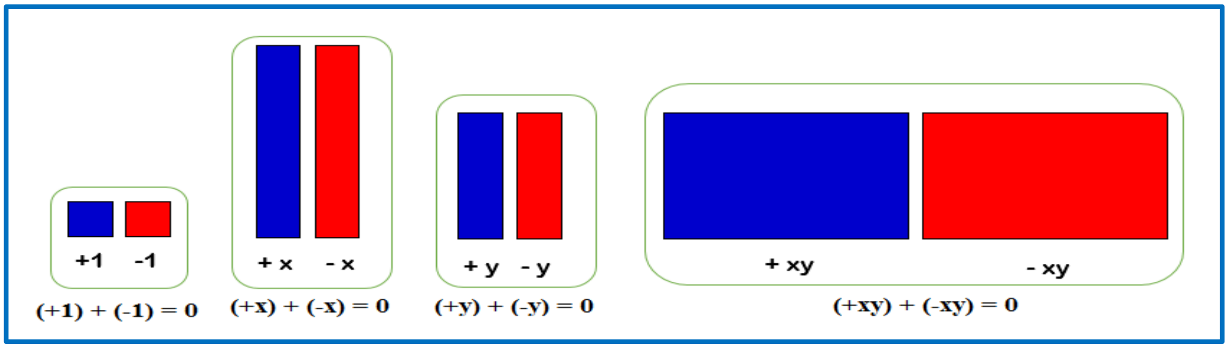
It is customary to use one colour to represent positive terms and a different colour to represent negative terms. Often the positive terms are white and the negative terms are darker but here **blue** and **red** are used for the indication of positive and negative.

We use two interpretations of the tiles in order to represent terms, the positive terms (blue tiles), and the negative terms (red tiles).



Zero Principle

Zero principles are the strategy to use pairs of variables with the opposite sign because they are additive inverses of each other. These pairs are also known as zero pairs because when put together, they cancel each other out to equate zero.



Some Solved Examples

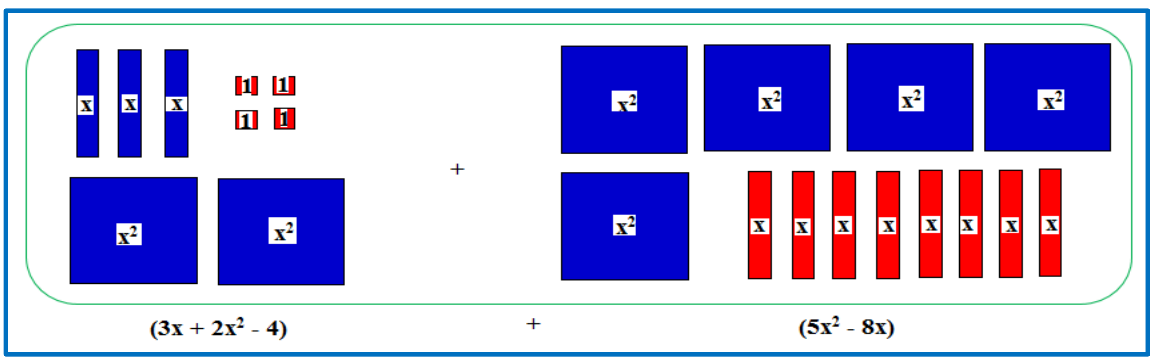
Example 1: Adding Polynomials using algebra tiles

Adding polynomials means putting them together by combining like terms.

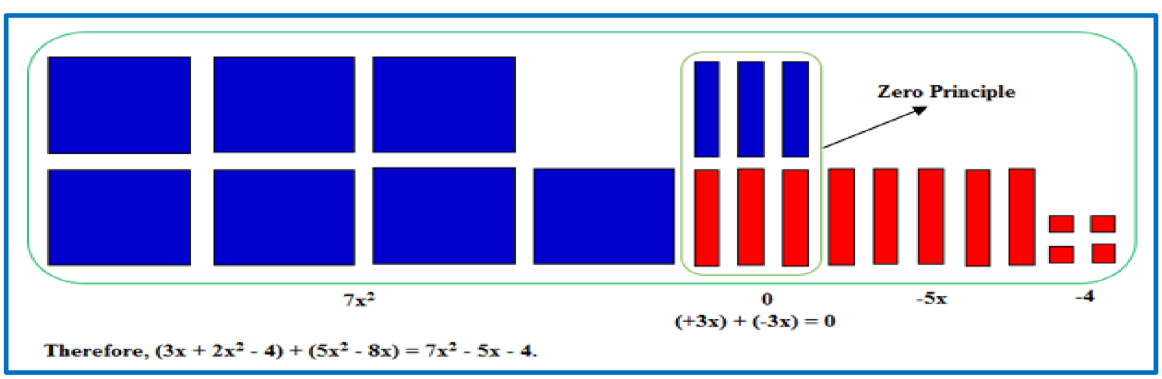
$(3x + 2x^2 - 4) + (5x^2 - 8x)$

Solution:

$$(3x + 2x^2 - 4) + (5x^2 - 8x)$$



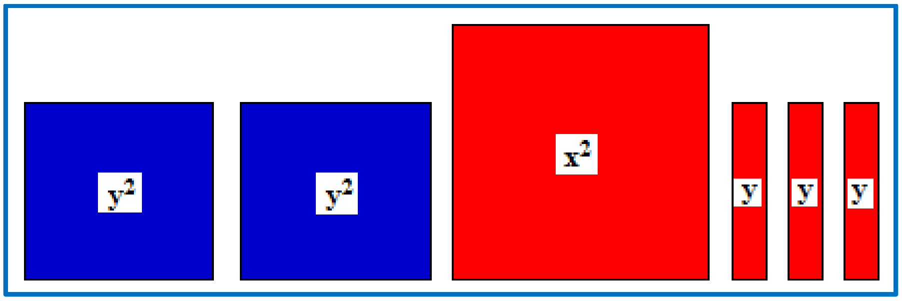
Re-arrangement: (Put the tiles together by combining like terms)



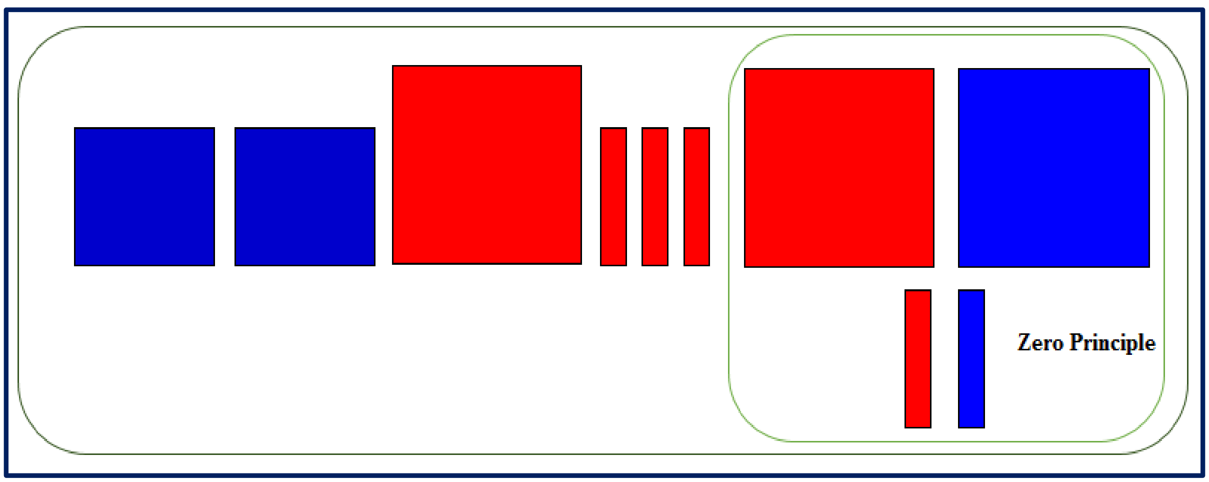
Example 2: Subtracting Polynomials using algebra tiles

$(2y^2 - x^2 - 3y) - (-2x^2 + y)$

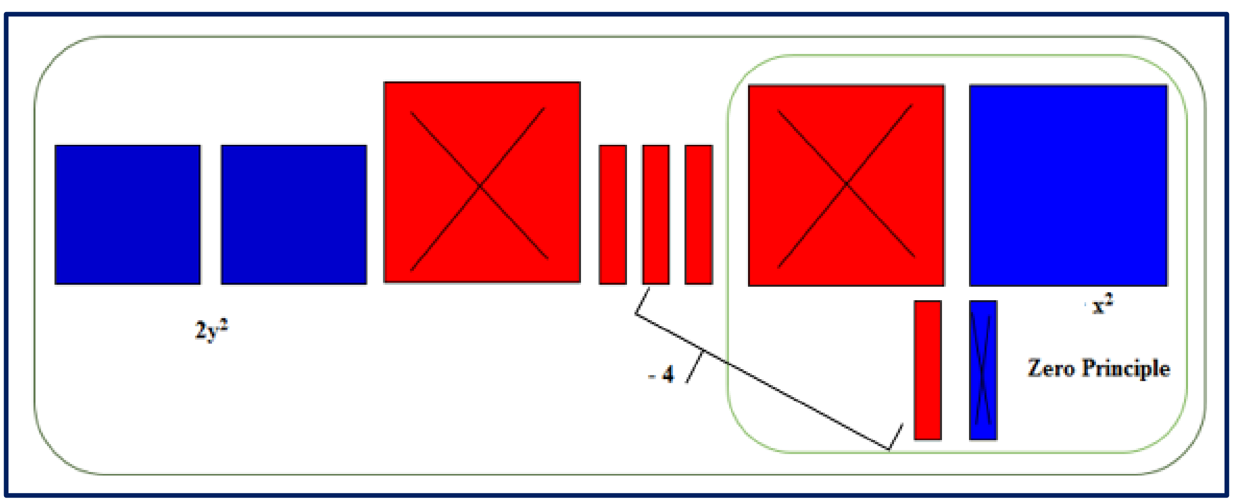
Solution:



Now, take away ($-2x^2$) and (+y) from the box above but, if you carefully look into the box, it's not possible to take away exactly. So, **zero principles** must be used to make things possible as shown below.



Now, it's possible to take away as shown below. The remainings algebraic tiles are the solutions.



Therefore, $(2y^2 - x^2 - 3y) - (-2x^2 + y) = 2y^2 + x^2 - 4y \text{ or } x^2 + 2y^2 - 4y$

Example 3: Adding and Subtracting Polynomials Symbolically

We can also add or subtract polynomials symbolically, without using the tiles. We still combine like terms. When you combine like terms, you are simplifying the polynomial.
For example;

a) Add $(-3y - 2xy + x^2) + (-y + 3xy + x^2)$

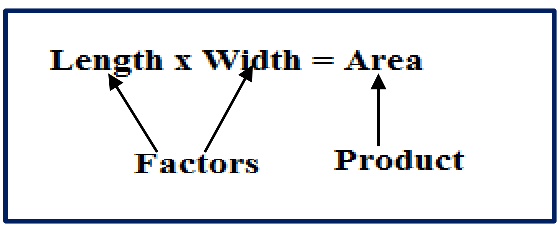
Solution:
$= (-3y - 2xy + x^2) + (-y + 3xy + x^2)$
$= [(-3y) + (-y)] + [(-2xy) + (3xy)] + [x^2 + \ x^2]$
$= -4y + xy + 2x^2$

b) Subtract $(-3y - 2xy + x^2) - (-y + 3xy + x^2)$

Solution:
$= (-3y - 2xy + x^2) - (-y + 3xy + x^2)$
$= (-3y - 2xy + x^2) + (+y - 3xy - x^2)$ [Subtracted by adding the opposites]
$= [(-3y) + (y)] + [(-2xy) + (-3xy)] + [x^2 - \ x^2]$
$= -2y - 5xy$

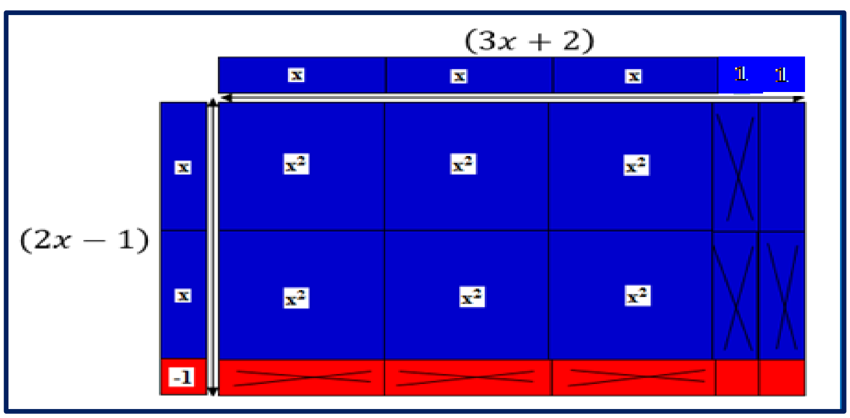
Example 4: Multiplying Polynomials using algebra tiles

To multiply two polynomials of degree 1, we use the area model whereby we create a rectangle whose dimensions are the two polynomials to be multiplied. The area of the rectangle in tiles represents the product:



For example, model the given multiplication and find the product: $(2x - 1)(3x + 2)$.

Therefore, $(2x - 1) \times (3x + 2) = 6x^2 + x - 2$.



Example 6: Multiplying Polynomials Symbolically

When we multiply two polynomials, each term in the first polynomial is multiplied by each term in the second polynomial and then the like terms are combined.

For example: $(2x - 1) \times (3x + 2) = 2x(3x + 2) - 1(3x + 2)$

$$= 6x^2 + 4x - 3x - 2$$

$$= 6x^2 + (4 - 3)x - 2$$

$$= 6x^2 + x - 2$$

Example 5: Dividing Polynomials using algebra tiles

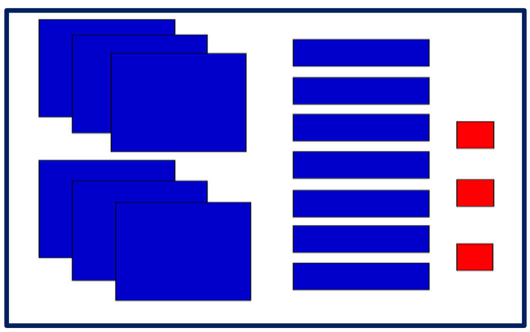
When we divide polynomials, we can use tiles and the area model. The area of a rectangle represents the dividend and one of the side lengths is the divisor. The other side length is the quotient.

Area ÷ side length = Unknown side length

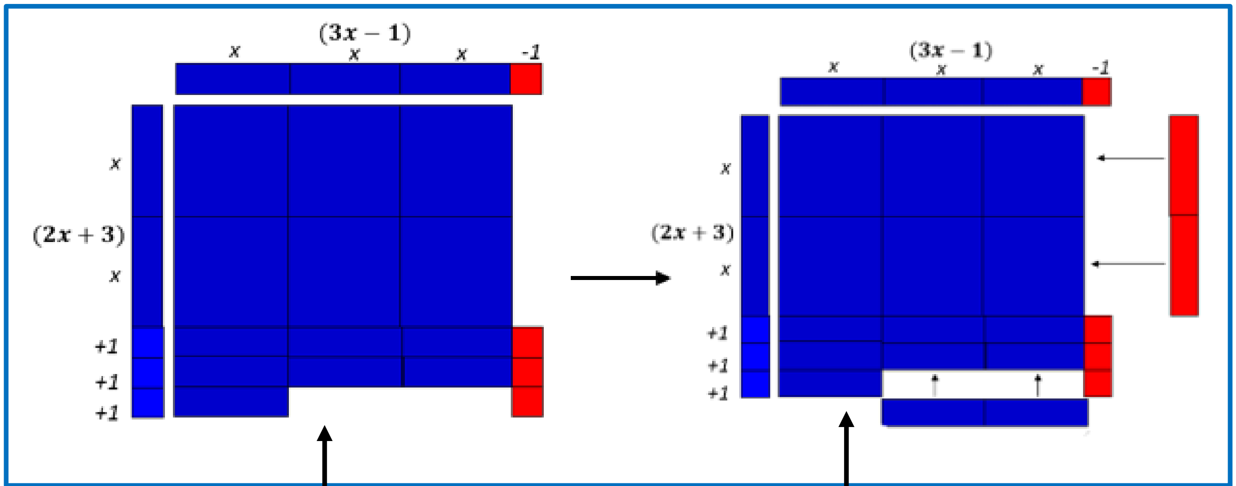
(Dividend ÷ divisor = Quotient)

For example, use algebra tile area model to help you divide $(6x^2 + 7x - 3) \div (3x - 1)$.

Step 1:
To divide $(6x^2 + 7x – 3) \div (3x – 1)$, gather together six x^2-tiles, seven x-tiles, and three –1-tiles.



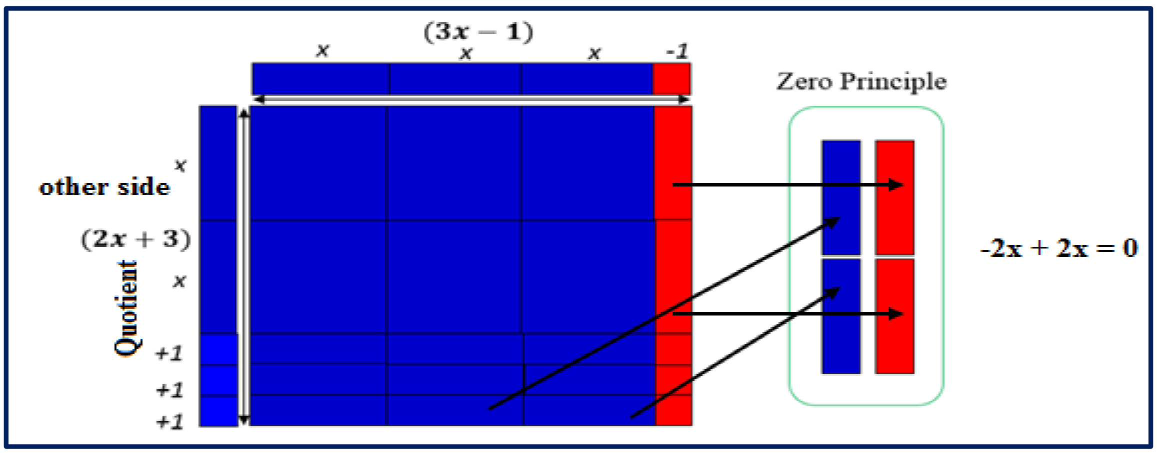
Step 2:
Arrange the tiles in a rectangle where one side has a length of $3x – 1$. The other side length will be the quotient.



It's not a rectangle yet, nor is the width $3x – 1$. You need two $–x$-tiles on the right side to make the width $3x – 1$.

If you add two $–x$-tiles to the right and two x-tiles to the bottom, you can make rectangle whose width is $3x – 1$ without changing the value of the area because $–2x + 2x = 0$.

Step 3: Create a complete area model or a perfect rectangle.



The quotient is the other side length, $2x + 3$.

Therefore, $(6x^2 + 7x – 3) \div (3x – 1) = 2x + 3$.

Example 6: Dividing Polynomials Symbolically (long division method)

You can also divide polynomials symbolically by thinking about how whole numbers are divided. When you calculate with whole numbers, you focus on the leftmost digits to give you an estimate, because those digits represent the most significant part of the number. You can use a similar approach to divide polynomials.

Old Division Form

Quotient

Divisor | Dividend

Remainder

Steps to divide polynomials symbolically:

Step 1:

Re-write the expression in the old division form.

For example:

$$x + 2\,\big)\overline{\,3x^3 + 6x^2 + x + 2\,}$$

Step 2:

Look for the highest power of x in the **divisor** and the highest power of x in the **dividend** and find how many times does the x in divisor go into that 3x cubed (3$\mathbf{x}^3$) in the dividend.

That's $3x^2$ times since x times $3x^2$ gives 3x cubed ($3x^3$). So, put that $3x^2$ up on top in quotient place and in line with the x^2 term. So, that's where it belongs.

$$\begin{array}{r l}
 & \quad 3x^2 \\
x + 2 & \big)\overline{\,3x^3 + 6x^2 + x + 2\,}
\end{array}$$

Step 3:

Multiply this $3x^2$ by each of these terms (x + 2) and place what you get directly underneath the polynomial in line with the x^3 term and x^2 term.

So, in this case, we have x times $3x^2$, which gives us 3x cubed, and we have $3x^2$ times 2, which gives us plus $6x^2$.

$$\begin{array}{r l}
 & \quad 3x^2 \\
x + 2 & \big)\overline{\,3x^3 + 6x^2 + x + 2\,} \\
 & \;\; 3x^3 + 6x^2
\end{array}
\qquad
\begin{aligned}
3x^2(x + 2) &= 3x^2 \times x + 3x^2 \times 2 \\
&= 3x^3 + 6x^2
\end{aligned}$$

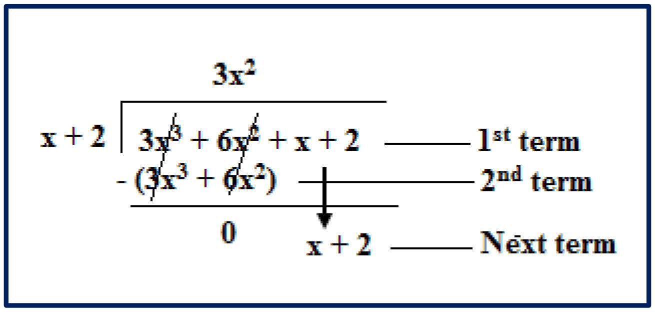
Step 4:

Subtract the 2nd line from the 1st line.

So, we have 3x cubed plus 6x squared ($3x^3 + 6x^2$) minus 3x cubed plus 6x squared. We get below zero ($3x^3 - 3x^3 = 0$) and we have $6x^2$ minus $6x^2$ that also gives us zero ($6x^2$ -$6x^2 = 0$).

$$\begin{array}{r l l}
 & \quad 3x^2 & \\
x + 2 & \big)\overline{\,3\cancel{x^3} + 6\cancel{x^2} + x + 2\,} & \text{——— 1st term} \\
 & \;- (\cancel{3}x^3 + \cancel{6}x^2) & \text{——— 2nd term} \\
\hline
 & \quad 0 &
\end{array}$$

Then carry down the remaining terms to the next term to the line.



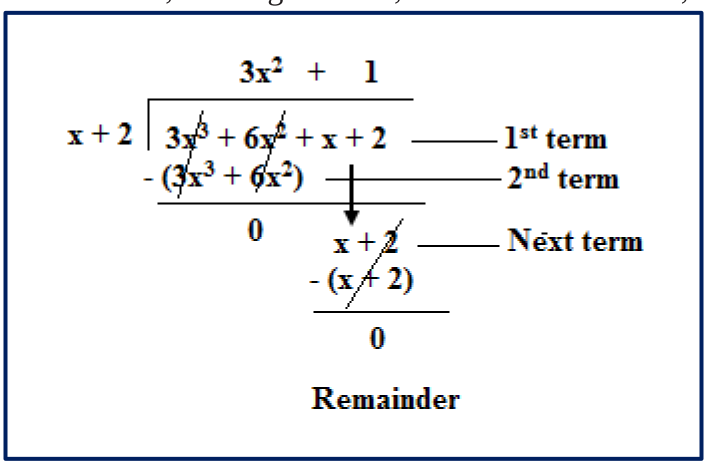
Step 5: Repeat step 2 & 3.

Look for the highest power of x in the **divisor** and the highest power of x in **dividend** and find how many times does the x in divisor go into that x in dividend (next term line).

That's 1 time since x times 1 gives x. So, put that 1 up on top in quotient place and in line with the constant term (2). So, that's where it belongs.

Multiply this 1 by each of these terms (x + 2) and place what you get directly underneath the polynomial in line with the x term and the constant.

So, in this case, we have x times 1, which gives us x, and we have 1 time 2, which gives us plus 2.

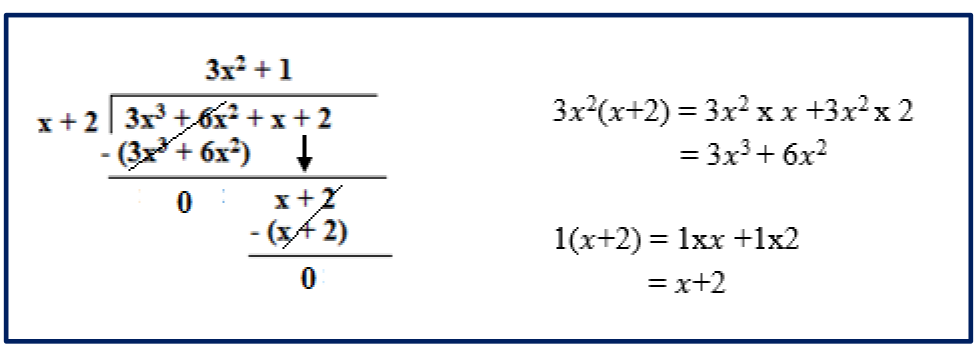


Therefore, $3x^3 + 6x^2 + x + 2$ by $x + 2 = 3x^2 + 2$.

The complete form would look like this.

For example, divide $3x^3 + 6x^2 + x + 2$ by $x + 2$.

Solution:



We can also divide polynomials in another way but with a similar approach. For example, to divide two numbers such as 1920 ÷ 60, you might estimate the quotient by dividing 1800 ÷ 60. Then you would multiply to get a product equal to the dividend, 1920.

1920 ÷ 60 =?

Estimate		Multiply	
$1920 \div 60 \approx 1800 \div 60 = 30$	$\longrightarrow$	$30 \times 60 = 1800$	
		$31 \times 60 = 1860$	
		$32 \times 60 = 1920$	32 works

1920 ÷ 60 = 32

You can use a similar approach to divide polynomials.

For example, to divide $3x^3 + 6x^2 + x + 2$ by $x + 2$,

- "Estimate" first by dividing $3x^3$ (in the dividend) by x (from the divisor). Just as with numbers, these leftmost terms are the greatest powers of each.
- Use your "estimate" to create a binomial with an unknown term.
- Multiply the binomial by the divisor.
- Use the coefficient of x or the value of the constant in the dividend to help you figure out the unknown term.

The coefficient of x in the dividend is 1, so Δ must be 1.

$(3x^3 + 6x^2 + x + 2) \div (x + 2) =?$

Estimate — Multiply

$(3x^3 + 6x^2 + x + 2) \div (x + 2)$

$\approx 3x^3 \div x = 3x^2 \longrightarrow (3x^2 + \Delta)(x + 2) = 3x^3 + 6x^2 + \Delta x + 2\Delta$

$= (3x^2 + \mathbf{1})(x + 2) = 3x^3 + 6x^2 + x + 2$

$(3x^3 + 6x^2 + x + 2) \div (x + 2) = 3x^2 + 1$

Note (s):

When working with polynomials of the same variable, the degree of the quotient is always the difference between the degrees of the dividend and the divisor. For example, when you divide a degree 4 polynomial in one variable by a degree 3 polynomial of the same variable, the quotient has degree 1, as shown below:

$$(3x^4 - x^3) \div x^3 = 3x - 1 \quad \text{degree 4} \div \text{degree 3} = \text{degree 1}.$$

This makes sense if you think of the exponent laws: $x^4 \div x^3 = x^1$.

Sometimes when you divide polynomials the quotient is not a binomial.

For example: $(x^3 - 1) \div (x - 1) = x^2 + x + 1$.

Activity 1

Solve the questions given below in your notebook.

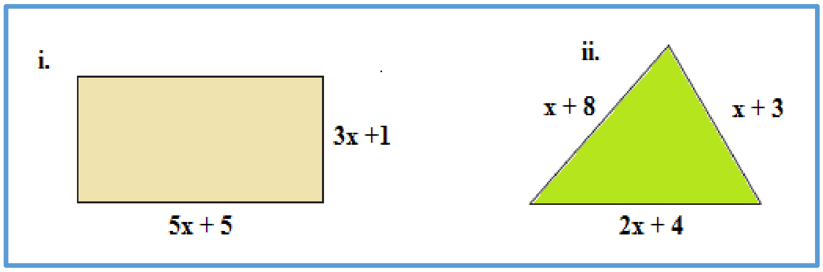
1.

a) Subtract using the zero principles.

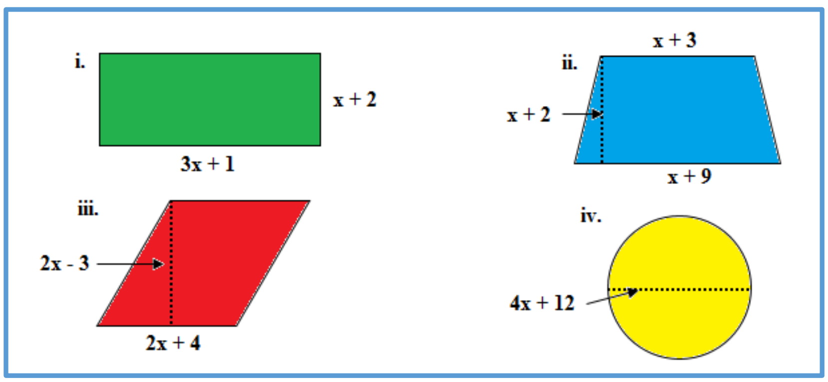
i. $(-3y + 2y^2 - 6x) - (3y + 4y^2 + x)$

ii. $(2y^2 - x^2 - 3y) - (-2x^2 + y)$

b) Express the perimeter of each as a polynomial. Remember to simplify.



2. Calculate the area of each shape.



3. Divide:

i. $(10x^2 + 19x + 6) \div (5x + 2)$

ii. $(6x^2 + x - 12) \div (2x + 3)$

Summary

- A **polynomial** is an algebraic expression that includes at least one variable.
- Each part of the polynomial separated by addition or subtraction signs is called a **term**.
- Polynomials with 1, 2, or 3 terms have special names.
- The **degree** of the polynomial is determined by the highest power.
- When a variable is multiplied by a number, whether an integer, a fraction, a decimal, or an irrational number like π, that number is called a **numerical coefficient**.
- Polynomials can have **variable terms** and **constant terms**.
- Sometimes polynomials include **like terms**. These are terms involving exactly the same variables raised to exactly the same powers.
- A polynomial can only be classified after all the like terms have been combined.
- You can **evaluate** a polynomial by substituting a value for the variables.
- Polynomials can be used to represent certain situations.
- Operations of polynomials can be done in two ways:
  1. Representation of algebra tiles (using an algebraic tiles)
  2. Symbolical interpretation (symbolically)
- It is customary to use one colour to represent positive terms and a different colour to represent negative terms. Often the positive terms are white and the negative terms are darker but here **blue** and **red** is used for the indication of positive and negative.
- **Zero principle** is the strategy to use pairs of variables with opposite sign because they are additive inverses of each other.
- These pairs are also known as zero pairs because when put together, they cancel each other out to equate zero.

Self-check for Learning

Solve the questions given below in your notebook.

Simplify the given polynomial both using the **algebraic tiles** and **symbolically**.

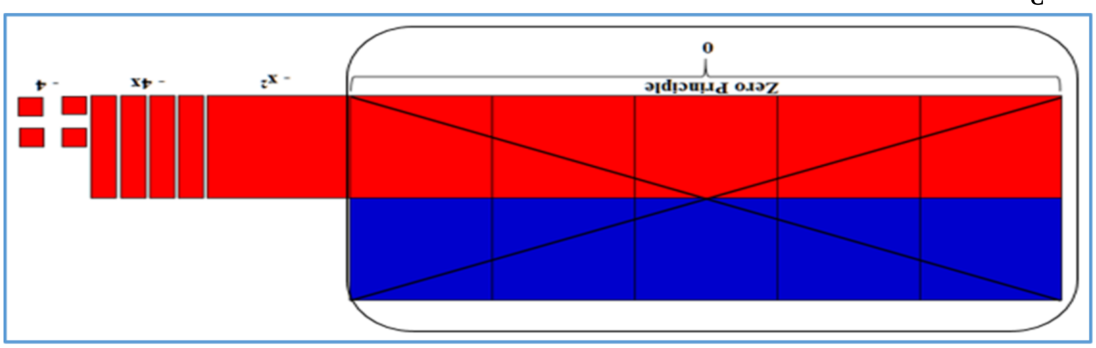
1. $(-2x - 6x^2 - 4) + (5x^2 - 2x)$
2. $(-3y + 2y^2 - 6x) - (3y + 4y^2 + x)$
3. $(2x + 1)(3x + 2)$
4. $(12x^2 + 5x - 2) \div (3x + 2)$

Self-check for Learning

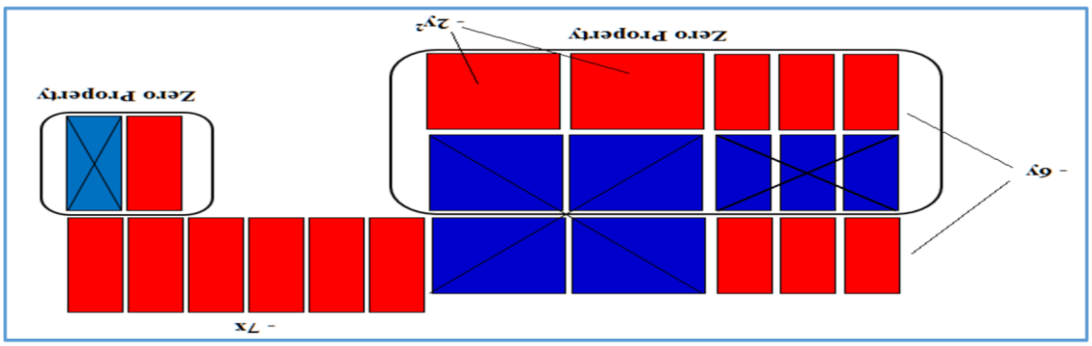
1. (Symbolically)
   1) $-4x - x^2 - 4$
   2) $-6y - 2y^2 - 7x$
   3) $6x^2 + 7x + 2$
   4) $4x - 1$

Algebraic tiles

1.

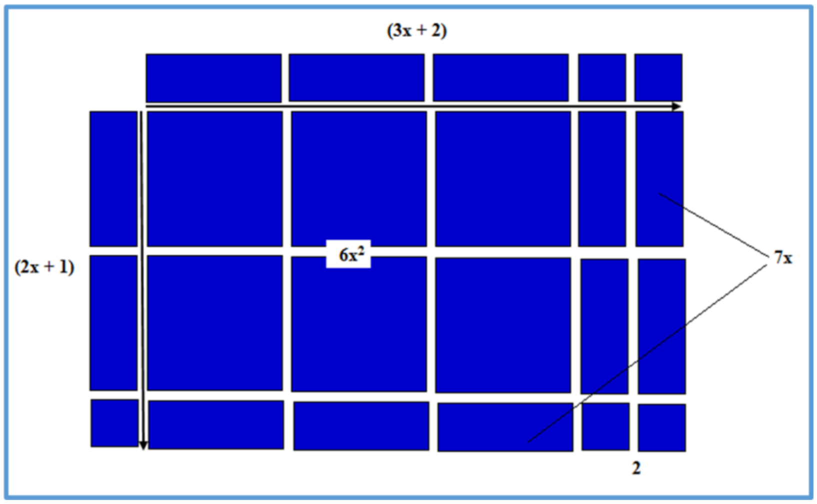


2.

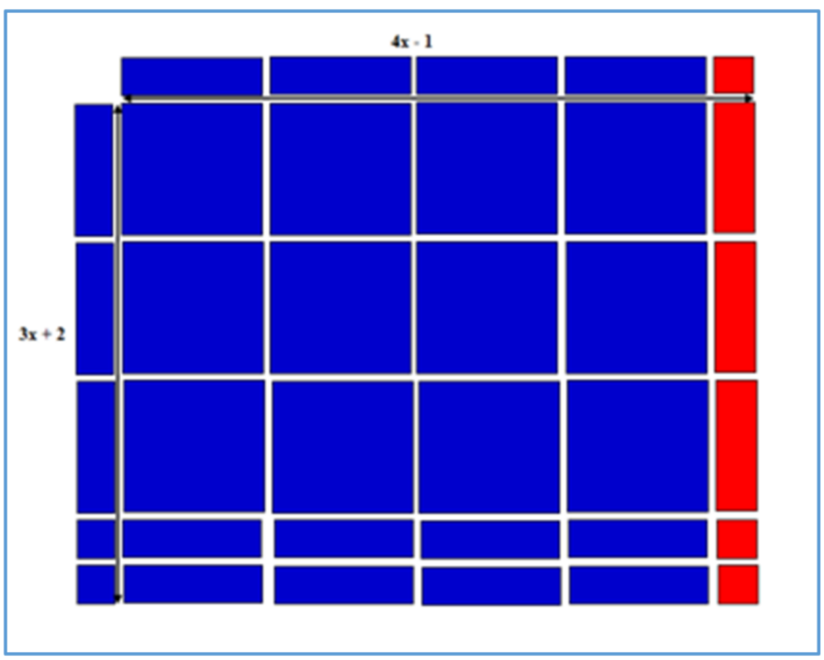


Self-check for Learning (Continued)

3.



4.



Lesson No: 1 **Subject:** Geography **Class:** 9 – 10 **Time:** 50 minutes

Topic: **The Growth of Industries**

1. Define the term industry.
2. Classify three sectors of industries with examples.
3. Differentiate between pre-modern and modern industries.
4. Explain the need for the establishment of modern industries in Bhutan.
5. Evaluate the impact of tourism industry in Bhutan.

Introduction

An **industry** is a group of manufacturers or businesses that produce goods or services. It produces a particular kind of goods or services and generates employment opportunities. Industry also includes other commercial activities that provide goods and services such as agriculture, transportation and hospitality.

What are three different sectors of industries.

Sectors of Industry

Generally, industries are classified into three sectors based on the nature of the activities as:

a. **Primary sector of industry**: Industries that have direct involvement in collecting or extracting raw materials. Examples of such industries are agriculture, forestry and mining. They are more prominent in developing countries.

b. **Secondary sector of industry**: Manufacturing industries that process raw material, causing it to undergo a physical change are called secondary industry. Agro-based industries like food processing, dairy products, distilling, mining and quarrying industry fall under the secondary sector.

c. **Tertiary sector of industry**: Industries that provide goods and services to people such as tourist operators, bankers, retail dealers, teachers and doctors.

Modern and Pre-Modern Industries in Bhutan

Industries that existed in Bhutan before 1950s are called pre-modern industries and the industries that were developed after the coronation of 3rd Druk Gyalpo to the golden throne in the year 1952 are called as modern industries in Bhutan. Construction of motor roads made it possible to transport heavy machineries that lead to the development of modern industries.

Differences between the Modern and Pre-Modern Industries

Modern Industries	Pre-modern Industries
➢ Goods are manufactured using machines.	➢ Goods are hand-made using simple tools.
➢ Produce all kinds of goods.	➢ Produce mainly agricultural and household goods.
➢ Produce goods in large quantities/scale.	➢ Produce a small quantity of goods.
➢ Goods are produced in a large or medium scale industry.	➢ Goods are produced in small or cottage industry.
➢ Employ more number of labour.	➢ Employ a few labour.
➢ Localized in a particular location/place.	➢ Found dispersed throughout the country.

List some of the items in your house that belonged to pre-modern industries and modern industries.

Common Pre- Modern Industries in Bhutan

a) Textile Works
b) Bamboo Works
c) Wood Works
d) *Desho* (Traditional Paper)
e) Iron and Gold Smithies

a. Textile works: Depending on the availabilities of raw materials, Bhutanese people engaged in various textile productions across the country. Home-based textile work has been a popular industry in Bhutan for a long time. Most materials used in weaving were available locally.


Home grown cotton in Chimoong, Pemagatshel©Tshering Phuntsho

People also raised silkworms in Bhutan and made cloth from raw silk called *bura* in Samdrup Jongkhar and Orong as shown in the picture below.

Despite having stiff competition from the factory-made clothes, traditional hand-woven cloth has maintained its great popularity throughout the world. It is mainly because of its beauty of the intricate patterns and uniqueness of the clothes.

b. Bamboo Works

People of *Kheng* use bamboo to make baskets, hats and other useful kitchen items. Skilled basket makers can be found in both foothills and the high-altitude regions. People in both the region barter their goods for cereals and other agricultural products with people from other parts of the country in the past.

c. Wood Works

Owing to the abundance of forest resources, wood work has always been an important craft of the Bhutanese. Different types of wood are used to make tools, containers and decorative items. Almost every farmer knows how to make the wooden tools needed on the farm. There are some very specialized crafts people in eastern Bhutan. They use lathes to make beautiful cups and bowls.

Cups called *dza-shing phob*, made from nodules found mainly in poplar, maple or wormwood, are very popular among Bhutanese people. *Dza-shing phobs* cup may cost as much as Nu. 5000 or more.

Other popular kinds of wood work are the decorative carvings which embellish buildings and the carving of xylographs or printing blocks.

d. *Desho* (Traditional Paper)

Desho is the traditional paper made in Bhutan. Paper making in this country is a living tradition. It is not known when and who introduced the skill. *Desho* making has now assumed a commercial scale in Thimphu. The product is also exported to a few countries like Japan and USA.

Desho is made from the bark of daphne, commonly known as *de shing*. It is a plant with fibrous bark growing at elevation between 1200 and 3000 metres above sea level and is found across the entire country.

Process of making *desho* paper

How is Bhutanese traditional paper made?

The picture 1-5 given below are the steps showing the process of desho making.

1. First, basts are removed from the Daphne plants and cut into short pieces.
2. The basts are soaked, cleaned and beaten into a soft pulp.
3. The pulp is mixed with water and then spread over the mat covered by cotton cloth.
4. The mat is then shaken from side to side until the pulp is evenly spread.
5. It is then dried in the sun and wind or over metal heater.

Did you know?

1. When Samuel Turner visited Bhutan in 1783, he found desho making a well-established tradition and he described desho as the strongest paper he had ever seen.
2. Unlike paper making industries in other countries the process of making desho does not use any chemicals that could cause environmental pollution.

What is *desho* paper used for?

Due to its strong fibre, *desho* is mainly used for important documents and religious scripts that needs to be preserved for ages. Now a days, it is also used for wrapping parcels and gifts. It can also be used for making bags, envelope, books, scrolls, greeting cards as shown in the next page.

e. Iron and Gold Smithies

A tradition of working with iron in Bhutan goes back many centuries. During the 8th century, *Sendha Gyap* of Bumthang is said to have built an iron castle at *Chag-khar* on the bank of Chamkhar chu.

In our country, crafts such as wood and metal working are considered professional occupations open to any person with sufficient interest and talent. They supplied utensils, tools, weapons and ornaments to various people according to their needs. The skills of these crafts people and their work continue to be in demand and are highly valued. The picture below shows some Bhutanese crafts.

A person who possessed the skills of all the crafts described above and other such as painting, sculpting and carpentry used to earn high regard in our society in the past. They are collectively known as *Zorig Chusum* (the 13 crafts). The 15thcentury saint Pema Lingpa and Zhabdrung Ngawang Namgyel in the 17th century were amongst the few who achieved all the 13 skills.

Why do you think industries have developed in Bhutan?

Reasons for the development of Industries in Bhutan

Modern industries have developed in Bhutan for the following reasons.

- To increase the people's level of income, which then increases their ability to purchase goods and services, leading to economic development.
- To encourage entrepreneurship among as many people as possible.
- To produce goods of good quality by using local materials, so that they can be sold abroad.
- To help reduce our dependence on imported goods.
- To create jobs for the unemployed people.

Distribution of Industries

There are several factors that have led to the concentration of industries in certain areas in Bhutan such as Phuntsholing, Passakha, Gedu, Tala, Thimphu, Gidakom, Dungsam and Samtse.

1. The national highway helped in transporting bulky machines and raw materials to the factory sites.
2. The hydroelectric power (HEP) stationed at Chukha has become a reliable source of power to run the machines in the factories.
3. The communication facilities developed in this region before they did in any other area, and this helped our industrialists to contact partners in different parts of the world.

Did you know?

The two largest industries in Bhutan are Bhutan Board Products Limited (BBPL) at Tala and Bhutan Carbide and Chemicals Limited (BCCL) at Pasakha, Phuntsholing.

Activity 1

List the factors that determine the location of an industry.

Service Industries

Service industries like shops, schools, banks, hospitals and service agents are growing quickly throughout Bhutan. One of Bhutan's most important service industries in terms of income is tourism. Tourism has become a vibrant business in Bhutan with a high potential for growth and further development. The Royal Government of Bhutan adheres strongly to a policy of 'High Value, Low Impact' tourism.

Tourism

Tourism comprises the activities of persons traveling to and staying in places outside their usual environment for not more than one consecutive year for leisure, business and other purposes. Travel has always been an important part of human activities. Tourism involves those activities undertaken by people who stay away from home for 24 hours or more on holiday. The development of tourism mainly depends on three factors:

- **Attractions**: Tourists are attracted to Bhutan's rich cultural and natural heritage. They come to Bhutan to trek through areas of pristine natural beauty as well as to learn about unique culture by visiting dzongs, villages, festivals and religious sites.

- **Accessibility**: Tourist must be able to get easily from their home country to the area they wish to visit for holidays or work. Today having Druk Air and Tashi airlines connected to few international airports, Bhutan has been made easily accessible to rest of the world.
- **Adequate service infrastructure**: This includes accommodation, food and transportation facilities. Today there are more than 200 travel agencies and 160 tourist hotels across the country registered with Tourism Council of Bhutan enhancing the service infrastructure.

Impact of Tourism

Tourism sector in Bhutan is regarded as one of the most exclusive travel destinations in the world. Bhutan's well-protected cultural heritage and natural environment are exemplary to the world. While tourism is a boon to the economy of a country, it has negative effects as well. Traffic congestions, air pollution, litter and rise in price for the local goods are on the list among the negative impact of tourism.

Activity 2

Tourism sector is one of the worst hit sectors due to COVID-19. Justify the statement with any three reasons.

Summary

- Industry can be categorized into primary, secondary and tertiary sectors.
- Developed countries have a majority of its population engaged in the secondary and tertiary industry.
- Tourism sector in Bhutan is growing rapidly. There is a major concern with this growth in terms of its impact on a country's environment, culture and traditions.

Self-check for Learning

Answers the questions given below in your notebook.

1. Why were the small manufacturing units dispersed throughout the country?
2. If you want to establish an industry, what are the key factors that you will take into consideration.
3. What are the positive and negative impacts of tourism?

Self-check for Learning

1. The small manufacturing units were dispersed throughout the country before the 1960s as Bhutan's economic development was limited to agriculture and the main function of the manufacturers was to provide tools and other household items of the farmers and their families. Some of the examples include gold smithies, paper makers, bamboo and wood artisans, weavers, carpenters.

2. The key factors that need to be taken into consideration while establishing any industry are

   a. Availability of raw materials.
   b. Availability of power.
   c. Cheap labour.
   d. Good transport network.
   e. Easy marketing and
   f. Suitable climate.

3. The positive and negative impact of tourism are:

Positive	Negative
✓ Hard currency revenue earner ✓ Provides employment and business opportunities ✓ It helps preserve national monuments and cultural heritage.	✓ Tourism, if uncontrolled can adversely affect the country's culture and tradition. ✓ It puts pressure on local resources and may impact the environment. ✓ Commodification of spiritual and religious objects and practices leading to degeneration.